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ASSESSING THE RELATIVE EFFICIENCY OF AIRCRAFT MAINTENANCE TECHNOLOGIES: AN APPLICATION OF DATA ENVELOPMENT ANALYSIS

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Abstract—This study focuses on discretionary maintenance strategies and their relationship to aircraft reliability, as measured by the percentage of scheduled flights delayed because of mechanical problems. The methodology of Data Envelopment Analysis is employed to identify the various strategies employed by the major airlines over the time period 1990–1994. Additionally, this methodology allows for a normative assessment as to which strategies are relatively efficient. Furthermore, the specific strategies utilized by efficient and inefficient airlines can be compared at a micro-level and thus quantifiable recommendations for the latter group can be suggested. © 1998 Elsevier Science Ltd. All rights reserved

1. INTRODUCTION

There has been much concern of late that rising costs and shrinking profit margins may have undesirable effects on the ability of airlines to maintain acceptable levels of safety performance. Specifically, such concern calls into question discretionary managerial strategies on the part of airline executives with regard to the performance of aircraft maintenance. The researcher is challenged to identify and quantify those discretionary strategies most likely to impact firm and industry levels of maintenance effectiveness. Furthermore, once the array of strategies is identified, there exists the additional research challenge of determining, in a meaningful manner, those strategies deemed to be most efficient.

Prior empirical work strongly suggests that airline safety has not declined since deregulation. Previous studies have utilized accident and passenger fatality rates in examining whether airline safety performance has deteriorated since deregulation. Rose (1989, 1992) in regressing the log of accident rates on a time trend variable found that "improvements in airline safety do not appear to have slowed appreciably since deregulation". She does caution that in the more recent time period, 1987–90, there is a tendency for accident rates to lie slightly above the long-term trend line, possibly suggesting that regulatory effects may operate with long lags.

Barnett and Higgins' (1989) calculations demonstrate that the fatality risk for a passenger on a domestic airline declined from an average of 1 in 2.5 million flights over the period 1971–1978 to 1 in 7.4 million flights over the period 1979–1986. If the researcher restricts the calculation to *established carriers*, the fatality risk over the latter period is 1 in 11.8 million flights. Oster and Zorn (1989) find, in comparing the periods 1970–1978 and 1979–1985, that accident rates due to pilot or controllers' errors, equipment failure and other aircraft declined in the deregulated period.

This study focuses on a more subtle issue. The question to be investigated is whether there are significant differences, for major airlines (part 121 carriers), in the *efficiencies of maintenance*

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technologies across airlines. Furthermore, one would also like to investigate the efficiency trends over time across airlines. The particular phenomena examined is the relationship between maintenance expenditure allocation strategies and the percentage of scheduled flight arrivals delayed because of mechanical problems. This allows for the exploitation of an extremely large data set in order to obtain a realistic and statistically meaningful view of the effectiveness of the various strategies identified in the sample used in this study.

Additionally, the study utilizes the methodology of Data Envelopment Analysis (DEA). This approach not only allows for the identification of particular strategies but also allows the researcher to make *normative* judgements as to the *relative efficiency* of any given strategy. Implications for discretionary managerial policies can thus be drawn.

2. METHODOLOGY

The theory and development of the Data Envelopment Analysis (DEA) model utilized is presented in detail in Appendix A. Nonetheless, some review of this technique facilitates a clearer understanding and interpretation of the results obtained here.

Much of traditional economic analysis, in examining the production efficiency of decision-making units, in a specific sample, makes *a priori* assumptions as to the mathematical form of the production function being utilized by these units. The specific parameters of the production function are estimated from the available sample thus yielding an average production function. Efficiency for each decision-making unit is measured in an absolute sense against this production function.

However, there are cases, such as in the case of airline maintenance activities, where the form of the production function is not known. In such cases, it may be appropriate to use DEA because it is a flexible, nonparametric technique that makes no assumptions about the form of the production function. Instead, it estimates an empirical 'best practice production frontier' from the observed inputs and outputs of individual units. This frontier replicates the behavior of *individual* units rather than that of the average sample estimate of conventional production functions. The DEA best practice frontier is necessarily piecewise linear and approximates the *true* production function. A decision-making unit is *efficient* when comparisons with other units indicate no inefficiency in the utilization of inputs and outputs, as measured by its position relative to the efficient production frontier.

In the particular case of maintenance expenditures for aircraft, an airline is operating with economic efficiency if it has found a combination of inputs that allows for the production of the desired level of aircraft reliability at minimum cost. Specifically, the orientation of the model employed in this study seeks to determine, for each airline, what proportional reduction in inputs is possible for a given level of aircraft reliability.

This latter notion is captured by the measure iota reported below. Intuitively, iota is the multiple of the vector of inputs that would yield the current level of aircraft reliability for a particular airline. For efficient airlines, iota is equal to one. Inefficient airlines will have values for iota of less than one—that is, some proportional reduction in inputs is possible.

Such analysis has interesting managerial implications. Suppose an airline, under consideration, has a peer group of airlines that have comparatively efficient aircraft reliability production techniques which allows them to achieve the same levels of reliability as the airline being examined, but more efficiently. If iota is very small, then the production technique of this airline is really off the mark. This indicates that the airline is primarily inefficient in the output of aircraft reliability. Attention should be focused on its output with consideration given to a shift of its input/production technique. If, on the other hand, iota is close to one, then this airline could remain with its current aircraft reliability production technique and achieve the same levels of aircraft reliability output with a small scaling down.

3. DATA SOURCES AND VARIABLE DEFINITIONS

This study included the major carriers possessing the necessary consistent data for the years 1990–1994. These included American, Continental, Delta, Northwest, Southwest, TWA, United and USAir. Air Alaska was removed from the sample because of inconsistencies and changes in accounting standards used by this airline. The output variable was defined to be the percentage of all scheduled flight arrivals not delayed for mechanical reasons. That is, this variable is one minus the percentage of scheduled flight arrivals delayed for mechanical reasons *not including weather or scheduling problems*. This was constructed as follows:

1. The number of scheduled flight arrivals by airport for each airline was obtained from Data Bank 28DS, T-100 Domestic Segment Data from the Department of Transportation,Office of Airline Statistics.
2. The total number of scheduled flight arrivals not including those delayed for mechanical reasons by airport for each airline was obtained from Air Travel Consumer Report from the Department of Transportation, Office of Consumer Affairs. A flight was reported as delayed if it operated more than 15 min after the scheduled time shown in the carriers' Computerized Reservation System.
3. The number of scheduled flight arrivals delayed for mechanical reasons was the difference between the above twomeasures. The actual output variable used was one minus thepercentage of all scheduled flight arrivals delayed for mechanical reasons.

Data for the input variables utilized in the study was obtained from Form 41 Financial Schedules, Data Bank 10, from the Department of Transportation, Office of Airline Statistics, Research and Special Programs Administration. The variables, representing all of the reported *non-overlapping* categories of maintenance expenses, were defined as follows:

1. Labor—airframes = labor expenses on airframes/total aircraft operating expenses
2. Labor—aircraft engines = labor expenses on aircraft engines/total aircraft operating expenses
3. Aircraft airframe repairs = expenditures on airframe repairs/total aircraft operating expenses
4. Aircraft engine repairs = expenditures on engine repairs/total aircraft operating expenses
5. Maintenance materials—airframes = material expenditures on airframes/total aircraft operating expenses
6. Maintenance materials—engines = material expenditures on engines/total aircraft operating expenses

The software utilized to perform the data envelopment analysis was the *Integrated Data Envelopment Analysis System: Version 5.1.*

4. RESULTS

Table 1 displays the relative efficiency (iota) results by airline, pooled by quarter, within each year. This sort of DEA 'window analysis' was performed to ensure that an airline, judged to be relatively efficient in a given year, was not deemed so because of uncontrollable external factors unique to a particular, limited time period. That is, an airline that is truly relatively efficient in a given year will be so regardless of the quarter selected. An airline that is relatively efficient only for a particular, limited time period must be viewed with some skepticism and caution. As will be discussed below, both of these scenarios did indeed emerge from the data.

Several observations can be made:

1. Delta, Southwest and USAir were on or very close to the efficient frontier during each of the five yearly periods. Each demonstrated consistency in their relative efficiency ranking from quarter to quarter.
2. American and United showed a need for a change in the production-input mix as well as the absolute level of inputs in 1990. Over time, both moved to the efficient frontier by remedying both these deficiencies. United accomplished this quite quickly, being on the frontier in all four quarters of 1991. There are some indications of inconsistency in this posture in late 1993 and early 1994. American took somewhat longer to achieve relative efficiency on the frontier, accomplishing this in 1994.
3. Continental, TWA and Northwest showed movement to the efficient frontier over time. There are clear indications of changes in production–input mixes and absolute levels of inputs. However, all three displayed considerable inconsistency in maintaining positions of relative efficiency.

Table 1. Airline efficiency scores by quarter

Airline and Quarter	Iota (90)	Iota (91)	Iota (92)	Iota (93)	Iota (94)
American (1)	0.63842	0.82388	0.86064	0.90298	1.00000
American (2)	0.58098	0.77131	0.85665	0.88473	1.00000
American (3)	0.66531	0.78140	0.86877	0.93252	1.00000
American (4)	0.77846	0.88734	0.95017	1.00000	NA
Cont'l. (1)	0.64276	0.78718	0.80433	1.00000	0.68946
Cont'l. (2)	0.55974	0.63774	0.80050	0.92032	0.88541
Cont'l. (3)	0.68399	0.71549	0.80807	1.00000	1.00000
Cont'l. (4)	0.73962	1.00000	0.83798	0.73069	NA
NA Delta (1)	1.00000	1.00000	1.00000	1.00000	1.00000
Delta (2)	1.00000	0.94845	1.00000	1.00000	1.00000
Delta (3)	1.00000	1.00000	1.00000	1.00000	1.00000
Delta (4)	1.00000	1.00000	1.00000	1.00000	NA
NA Northwest (1)	0.69962	0.74145	0.83973	0.94148	0.79645
Northwest (2)	0.60717	0.75930	0.77521	0.87068	0.80841
Northwest (3)	0.65842	0.83310	0.98768	0.83522	0.77764
Northwest (4)	0.77850	0.76223	1.00000	0.93548	NA
Southwest (1)	0.87358	1.00000	1.00000	1.00000	1.00000
Southwest (2)	0.93182	1.00000	1.00000	1.00000	1.00000
Southwest (3)	1.00000	1.00000	1.00000	1.00000	1.00000
Southwest (4)	1.00000	1.00000	1.00000	1.00000	NA
TWA (1)	0.78588	0.95728	1.00000	0.80341	0.86497
TWA (2)	0.75516	0.90832	0.73159	1.00000	0.76861
TWA (3)	0.77465	0.89792	0.78797	0.86185	1.00000
TWA (4)	0.76841	0.79409	0.81326	0.79033	NA
United (1)	0.62803	1.00000	1.00000	1.00000	0.93685
United (2)	0.65236	1.00000	1.00000	1.00000	0.85602
United (3)	0.72708	1.00000	1.00000	0.90471	1.00000
United (4)	1.00000	1.00000	1.00000	1.00000	NA
USAir (1)	0.88454	1.00000	1.00000	0.96759	0.93532
USAir (2)	0.76840	1.00000	1.00000	1.00000	1.00000
USAir (3)	0.88074	1.00000	1.00000	1.00000	1.00000
USAir (4)	1.00000	1.00000	1.00000	1.00000	NA

4. If one compares 1990 and 1994, more airlines in the latter year have achieved the efficient frontier.

Table 2 catalogues the areas of input inefficiency for the group of relatively inefficient airlines in each period. A general pattern seems to emerge. When an airline behaves relatively inefficiently, it does not do so with regard to a particular input category. Rather, it would tend to display inefficiencies across multiple input categories.

This observation is also consistent with the values reported for iota in Table 1. Recall that a value for iota significantly less than one would suggest that an airline's production technique for maintenance efficiency requires considerable adjustment (i.e. output inefficient). This phenomena is probably, to a large extent, what is being reflected in the pattern of Table 2.

Table 3 focuses more specifically on the input category differences between efficient and inefficient firms. Again several observations can be made:

1. Inefficient airlines consistently spent more than their relatively efficient counterparts on labor associated with both airframes and engines.
2. Inefficient airlines also consistently spent more than their relatively efficient counterparts on materials associated with airframes and engines.
3. Except for 1990, efficient airlines spent more than their relatively inefficient counterparts on repairs associated with airframes and engines.

Table 2. Incidence (by category) of input inefficiency by year (inefficient airlines)

	1990 (%)	1991 (%)	1992 (%)	1993 (%)	1994 (%)
Labor—airframes	21 (87.5)	12 (75.0)	8 (57.1)	6 (50.0)	6 (60.0)
Labor—engines	15 (62.5)	9 (56.3)	3 (21.4)	0 (00.0)	0 (00.0)
Repairs—airframes	15 (62.5)	7 (43.8)	2 (14.3)	2 (16.7)	4 (40.0)
Repairs—engines	14 (58.3)	7 (43.8)	6 (42.9)	6 (50.0)	5 (50.0)
Materials—airframes	22 (91.75)	10 (62.5)	13 (92.9)	6 (50.0)	5 (50.0)
Materials—engines	17 (70.8)	11 (68.8)	4 (28.6)	4 (33.3)	5 (50.0)

Table 3. Mean values for input variables efficient vs inefficient airline groups

	Efficient (%)				
	1990	1991	1992	1993	1994
Labor—airframes	2.4	2.9	2.8	3.3	3.0
Labor—engines	0.6	0.7	0.8	0.8	0.8
Repairs—airframes	1.7	2.1	3.8	3.2	2.8
Repairs—engines	3.0	4.4	2.5	3.5	3.9
Materials—airframes	2.5	2.7	2.9	2.7	2.3
Materials—engines	1.6	1.7	2.2	2.1	2.2
	Inefficient (%)				
Labor—airframes	3.4	3.6	3.9	3.6	3.8
Labor—engines	1.1	1.3	1.1	1.2	1.0
Repairs—airframes	1.9	1.6	1.9	1.8	2.1
Repairs—engines	3.1	2.2	2.8	2.8	3.2
Materials—airframes	3.4	3.6	3.7	3.0	3.0
Materials—engines	2.9	3.6	3.2	3.1	3.4

Recalling that the input variables represent all the non-overlapping categories of reported maintenance expenses the following would appear to be a fair summary of the above results: Efficient airlines spent more on the primary activity of effecting the repairs necessary to keep aircraft airworthy. Inefficient airlines performed less actual repair activities but, in fact, spent more on the labor and materials costs of this primary activity. Furthermore, as shown in Table 2, there seems to be evidence that relative inefficiency with regard to maintenance is a function of not only improper levels of input utilization but also improper choices with regard to actual production techniques.

It was noted above that a DEA 'window analysis' was performed to ensure that the normative judgement of relative efficiency for a particular airline was not due to uncontrollable external factors unique to a particular time period. The value of this approach is seen in the cases of Continental and Northwest. It was noted above, that for isolated periods Continental and Northwest did achieve the efficient frontier. However, there is a lack of consistency in these positions. One plausible explanation for this is that, in terms of the weighted average of aircraft, these two airlines have the oldest fleets. This would have the effect of imposing constraints on these airlines, relative to their competitors, with regard to the maintenance technologies available to them. This possibility is investigated below.

An interesting issue that needs further illumination is whether those airlines deemed relatively efficient, and thus on the efficient frontier, are also the best performing in terms of delayed scheduled arrivals due to mechanical problems. Table 4 investigates this issue. In the majority of cases, airlines on the efficient frontier are above average in terms of maintenance performance, that is, *the percentage of scheduled arrivals delayed because of maintenance problems is below the average for the entire sample*.

One noteworthy exception is Southwest. While consistently on the efficient frontier, Southwest, except for one quarter, is below average in terms of maintenance performance. Of all the airlines in the sample, Southwest expends the least amount on aircraft operating expenses. Thus, Southwest, operating with limited resources, is maximizing its maintenance performance given its financial limitations.

5. THE FLEET AGE FACTOR

While the above results are quite revealing, the question naturally arises as to the relationship between airfleet age and the degree of managerial discretion with regard to maintenance allocation decisions. Consistent data, to investigate this question, was available for all the airlines in the sample for 1994. Prior to 1994, Northwest was privately held and therefore much of the necessary data was not reported. Furthermore, the manner in which information, on airfleet age, was reported in prior years was not necessarily done in a consistent manner. The information in Table 5, obtained from 10K reports, *Moody's Transportation Manual*, and interviews with the airlines themselves, presents the weighted averages for airfleet age.

Utilizing Pearson Correlation Coefficients, an analysis was performed to investigate whether any statistically significant relationship existed between the managerial discretionary variables utilized in this study and airfleet age. The results in Table 6 demonstrate that, in this regard, no

Table 4. Relationship between position on efficient production frontier and performance

Eff. observation	Maintenance perf.	Eff. observation	Maintenance perf.
American 93,4	Above average	Southwest 93,1	Below average
American 94,1	Below average	Southwest 93,2	Below average
American 94,2	Above average	Southwest 93,3	Below average
American 94,3	Above average	Southwest 93,4	Below average
Continental 91,4	Above average	Southwest 94,1	Below average
Continental 93,1	Below average	Southwest 94,2	Below average
Continental 93,3	Above average	Southwest 94,3	Below average
Continental 94,3	Below average	TWA 92,1	Below average
Delta 90,1	Above average	TWA 93,2	Above average
Delta 90,2	Above average	TWA 94,3	Above average
Delta 90,3	Above average	United 90,4	Above average
Delta 90,4	Above average	United 91,1	Below average
Delta 91,1	Above average	United 91,2	Above average
Delta 91,3	Above average	United 91,3	Above average
Delta 91,4	Above average	United 91,4	Above average
Delta 92,1	Above average	United 92,1	Above average
Delta 92,2	Above average	United 92,2	Above average
Delta 92,3	Above average	United 92,3	Above average
Delta 92,4	Above average	United 92,4	Above average
Delta 93,1	Above average	United 93,1	Below average
Delta 93,2	Above average	United 93,2	Above average
Delta 93,3	Above average	United 93,4	Below average
Delta 93,4	Above average	United 94,3	Above average
Delta 94,1	Above average	USAir 1990,4	Above average
Delta 94,2	Above average	USAir 1991,1	Above average
Delta 94,3	Above average	USAir 1991,2	Above average
Northwest 92,4	Above average	USAir 1991,3	Above average
Southwest 90,3	Below average	USAir 1991,4	Above average
Southwest 90,4	Below average	USAir 1992,1	Above average
Southwest 91,1	Below average	USAir 1992,2	Above average
Southwest 91,2	Below average	USAir 1992,3	Above average
Southwest 91,3	Above average	USAir 1992,4	Above average
Southwest 91,4	Below average	USAir 1993,2	Above average
Southwest 92,1	Below average	USAir 1993,3	Above average
Southwest 92,2	Below average	USAir 1993,4	Above average
Southwest 92,3	Below average	USAir 1994,2	Above average
Southwest 92,4	Below average	USAir 1994,3	Above average

Table 5. Fleet age (weighted average)—1994

Airline	Fleet age—weighted average
American	8.0
Continental	17.2
Delta	9.8
Northwest	16.8
Southwest	7.6
TWA	12.0
United	10.0
USAir	10.4

Table 6. Correlation analysis—1994 (levels of statistical significance in parentheses)

	LABAFR	LABENG	REPAFR	REPENG	MATAFR	MATENG	AGE
LABAFR	1.00000						
	(0.0000)						
LABENG	0.54950	1.00000					
	(0.1583)	(0.0000)					
REPAFR	−0.13068	−0.57641	1.00000				
	(0.7577)	(0.1348)	(0.0000)				
REPENG	−0.22622	−0.64354	0.98372	1.00000			
	(0.5901)	(0.0851)	(0.0001)	(0.0000)			
MATAFR	−0.21647	−0.50938	0.02045	0.15921	1.00000		
	(0.6066)	(0.1973)	(0.9617)	(0.7065)	(0.0000)		
MATENG	−0.01070	0.45426	−0.81947	−0.77084	−0.08783	1.00000	
	(0.9799)	(0.2582)	(0.0128)	(0.0252)	(0.8362)	(0.0000)	
AGE	0.21673	−0.04470	−0.04354	0.05281	0.61024	0.29003	1.00000
	(0.6062)	(0.9163)	(0.9185)	(0.9012)	(0.1081)	(0.4859)	(0.0000)

significant relationship was found. Furthermore, the significant relationships that do emerge are consistent with the above data envelopment analysis.

6. CONCLUSIONS

This study has pursued an analysis of identifying those discretionary managerial strategies undertaken by airlines with regard to aircraft maintenance. In addition, it has utilized the normative procedure of DEA to assess which of these strategies were relatively efficient.

The results show that, for airlines to achieve the efficient frontier, a twofold process was required. First, those airlines, not already on the efficient frontier, typically needed to pursue a change in the production–input mix. Second, a further change in the absolute level of inputs was required. This suggests that discretionary managerial strategies with regard to maintenance expenditure allocations required sophisticated formulations.

Significantly, by 1994 most of the major airlines in this study had achieved the efficient frontier. Recent activities demonstrate that airlines are aggressively pursuing new and innovative strategies with regard to aircraft maintenance. USAir uses its IMS system, an integrated maintenance system which in fact is an earlier version of products it markets to other carriers, Merlin and Maxi Merlin. In 1992 Delta began implementing its MARC (Maintenance and Rebuild Control) project. Furthermore, an interesting problem will confront airlines as they need to assimilate cutting edge technologies, as embodied in the Boeing 777, into air fleets of various vintages.

REFERENCES

Ali, A. and Seiford, L. (1993) The mathematical programming approach to efficiency analysis. In *The Measurement of Productive Efficiency: Techniques and Applications*, eds H. Fried, C. Lovell and S. Schmidt. Oxford University Press, New York.

Barnett, A. and Higgins, M. (1989) Airline safety: the last decade. *Management Science* **35** (1), 1–21.

1 Consulting (1995) *Integrated Data Envelopment Analysis System: Version 5.1.* Amherst, MA.

Oster, C. V., Jr. and Zorn, C. K. (1989) Airline deregulation: is it still safe to fly? In *Transportation Safety in an Age of Deregulation*, eds L. Moses and I. Savage. Oxford University Press, Oxford.

Rose, N. L. (1989) Financial influences on airline safety. In *Transportation Safety in an Age of Deregulation*, eds L. Moses and I. Savage. Oxford University Press, Oxford.

Rose, N. L. (1992) Fear of flying? economic analyses of airline safety. *Journal of Economic Perspectives* **6** (2), 75–94.

U.S. Department of Transportation (1990–1994) *Air Travel Consumer Report*, Office of Consumer Affairs, Washington, DC.

U.S. Department of Transportation (1990–1994) *Data Bank 28DS: T-100 Domestic Segment Data*. Office of Airline Statistics, Data Administration Division DAI-20, Washington, DC.

U.S. Department of Transportation (1990–1994) *Form 41 Financial: Data Bank 10*. Office of Airline Statistics, Data Administration Division DAI-20, Washington, DC.

APPENDIX A

A1. DEA METHODOLOGY

The analysis in this study employed the input-oriented data envelopment model as specified by Ali and Seiford (1993). Using their notation, consider the case of n airlines, each utilizing, in varying amounts, m distinct safety-related inputs in order to produce s different safety-performance outputs. The objective of DEA can be specified so as to minimize total waste. Mathematically, this can be represented as:

$$\min_{\lambda_j, s_r, e_i}\left(\sum_{r=1}^{s}\mu_{ri}s_r + \sum_{i=1}^{m}\nu_{il}e_i\right) \tag{A1}$$

The variable s_r is the amount of slack in, or foregone amount of safety-performance output r, while the variable e_i is the excess amount of safety-related input I utilized. The values μ_{rl} and ν_{il} are shadow prices, or the marginal value of a unit of output or input. The analysis utilized in this study specified Airline Specific Bounds on the values of μ_{rl} and ν_{il} defined by:

$$\mu_{rl} = \frac{1}{y_{rl}},\, r = 1, \ldots, s \text{ with } \nu_{il} = \frac{1}{x_{il}},\; i = 1, \ldots, m \tag{A2}$$

This procedure allows the projections and efficiency scores derived to be independent of the units of measurement for the data, i.e. *units-invariant*.

This is a linear programming problem and the associated resource constraints and convexity conditions, with regard to input x_{ij} and output y_{rj} can be stated as:

$$\sum_{j=1}^{n} y_{rj}\lambda_j - s_r = y_{rl} \; r = 1, ..., s \; l = 1, ..., n \tag{A3}$$

$$-\sum_{j=1}^{n} x_{ij}\lambda_{ij} - e_i = -x_{il} \; i = 1, ..., m \; l = 1, ..., n \tag{A4}$$

$$\sum_{j=1}^{n} \lambda_j = 1 \tag{A5}$$

$$\lambda_j \geqslant 0 \; j = 1, ..., n \tag{A6}$$

$$s_r \geqslant 0 \; r = 1, ..., s \tag{A7}$$

$$e_i \geqslant 0 \; i = 1, ..., m \tag{A8}$$

The solution to the above problem identifies, for each airline, l, a projected point on the efficient frontier, $(\hat{x}_l - \hat{y}_l)$. Constraint (A5) determines the projected point as a convex combination (convex hull) of efficient airlines via the n-vector λ. This allows for variable returns to scale. The items x and y are the vectors of inputs and outputs. The essence of the efficiency evaluation of a particular airline (with an actual achieved combination of x_l and y_l) is the identification of excesses in input utilization $(x_l - \hat{x}_l)$ and deficiencies in output $(\hat{y}_l - x_l)$. A particular airline is deemed efficient if $(x_l, y_l) = (\hat{x}_l, \hat{y}_l)$, the airline thus lying on the efficient frontier. Thus, one possible measure of inefficiency, Δ^l, can be defined by:

$$\Delta^l = \mu^l(\hat{y}_l - y_l) + \nu^l(x_l - \hat{x}_l) \tag{A9}$$

Notice, that for efficient airlines $\Delta^l = 0$.

The model derived above is referred to as the *base model*. It embodies an approach consistent with a 'systems-oriented' philosophy where input and output variables are simultaneously determined. An alternative representation of the airline maintenance efficiency process would probably focus on a particular set of variables directly under managerial (discretionary) control(in this case input), yielding alternative projected points. More succinctly, in an input-oriented model, the researcher is able to investigate what proportional reduction in input(s) is possible for a given level of output(s). That is, an airline is operating with economic efficiency if it has found a combination of inputs that allows for the production of the desired level of aircraft reliability at minimum cost.

The notion of a projected point on the efficient frontier, as discussed above, can be used to illustrate the nature of oriented models. Typically, for a given airline l, the movement from (x_l, y_l) to $(\hat{x}_l, \hat{y}_l)$ can be thought of as a combination of two components. The first is a proportional change in output augmentation and input reduction. The second is a set of additional (nonproportional) residual output augmentation and input reduction after the initial proportional changes have taken place. Thus:

$$\hat{y}_l - y_l = \rho y_l + \delta_l^0 \tag{A10}$$

$$x_l - \hat{x}_l = \upsilon x_l + \delta_l^I \tag{A11}$$

As Ali and Seiford (1993) note, it is desirable from a theoretical and intuitive point of view, to separate out the proportional components in that they maintain the original 'technical' mixture between inputs and outputs.

For airline l, the output vector can be increased proportionally (in each vector component) by a factor of ρ with *individual nonproportional* residual component increases in each of the separate output variables given by δ_l^0. Similarly, the input vector can be decreased proportionally (in each vector component) by a factor of γ with *individual nonproportional* residual component decreases in each of the separate input variables given by δ_l^I. Therefore eqn (A9) now becomes:

$$\Delta^l = \rho(\mu^l y_l) + \gamma(\nu^l x_l) + \mu^l \delta_l^0 + \nu^l \delta_l^I \tag{A12}$$

Quite simply, the input-oriented model seeks to maximize γ.

This effectively identifies the intermediate point $(y_l, (1-\gamma)x_l)$ in the input-orientation case and for ease of notation, consider:

$$\theta = 1 - \gamma \tag{A13}$$

Thus, maximizing γ is equivalent to minimizing θ.

In the linear programming problem, in the input-oriented case, the input constraint is now replaced by:

$$-\sum_{j=1}^{n} x_{ij}\lambda_j + \theta^1 X_{il} - \delta_{il}^I = 0 \; i = 1, ..., m \; l = 1, ..., n \tag{A14}$$

Effectively, the input-oriented model requires the solution of the linear program with regard to the intermediate point $(y_l, \theta^l x_l)$ in order to obtain the projection point. The projected point obtained with this orientation can differ from that obtained from the base model.

In the base model case, Δ^1 was utilized as a measure of efficiency. However, in the cases of the input-oriented model Δ^l is not optimized. Therefore, an alternative measure of efficiency is required. One such measures is ι^l. Intuitively, $\iota;^l$ is the multiple of the input vector that would yield the current level of output for airline l. Thus:

$$l^1 = \frac{\mu^1 Y_1 + \omega^l}{\nu^l X_1} \tag{A15}$$

It follows that for efficient airlines, in the input-oriented case, $\iota^l = 1$.

A2. The base vs input-oriented model

As the orientation of the projection is changed, alternate projected points for inefficient airlines are obtained. These projected points are a reflection of the particular priority of a given orientation. The input orientation model seeks a projected point such that the proportional reduction in inputs is maximized (the role of θ). The implicit underlying premise in such an orientation is that the primary objective of the airline under evaluation is to gain efficiency by reducing excess input utilization while continuing to operate with its current technology mix (reflected in actual input ratios). A most desirable aspect of the input-oriented model is that because it measures inefficiency in terms of proportional changes in inputs, it allows an airline to be evaluated with respect to a best practice airline that is most similar to it in terms of input and output mixes.

Iota interprets the entire inefficiency of an airline in terms of input reduction. Since the projected point obtained, both in the nonoriented (base) and input-oriented cases, prescribes changes in both inputs and outputs, iota does not directly correspond to the 'ideal prescription'. However, it does convey information with regard to managerial policy as described in the text above.